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FREE-CONVECTION HEAT TRANSFER ON A VERTICAL SURFACE WITH A TEMPERATURE DISCONTINUITY

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Parametric correlations are obtained for calculating heat transfer on a vertical surface with a temperature discontinuity, over a wide range of variation of Prandtl number and for calculating the relations of temperatures at the wall.

The investigation of free convection heat transfer at a wall with various boundary conditions involves problems of singular perturbations of the full Navier—Stokes equations and the energy equation. It has been shown by the method of matched asymptotic expansions that in the first approximation this problem can be considered using the boundary-layer equations [1]. Numerical calculations of free convection on a vertical surface in air [2, 3] agree well with experimental data [4]. The available data in the literature on heat transfer refer to particular cases of temperature discontinuity and to Pr = 0.7.

We consider free convection on a vertical plane surface. On the lower part of the wall ($0 \leqslant x \leqslant x_0$) the temperature is given as T_{W1}, and on the upper part ($x > x_0$) the temperature is T_{W2} ($T_{W1} > T_\infty$, $T_{W2} > T_\infty$). Due to the temperature difference between the wall and the surrounding medium, the motion of the fluid is directed upward, parallel to the wall. Assuming that energy dissipation and the work of compression are negligibly small, we can represent the system of equations of motion and heat transfer in the boundary layer in the form [1]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\quad u\frac{\partial \vartheta}{\partial x}+v\frac{\partial \vartheta}{\partial y}=a\frac{\partial^2\vartheta}{\partial y^2},$$
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta\vartheta+\nu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

with the boundary conditions

$$u=0,\ v=0\ \text{for}\ y=0;\ u=0,\ \vartheta=0\ \text{for}\ y\to\infty;$$
$$\vartheta=\vartheta_{c1}\ \text{for}\ x\leqslant x_0;\ \vartheta=\vartheta_{c2}\ \text{for}\ x>x_0. \tag{2}$$

We now transform the system of equations (1), introducing the stream function $\Psi(x, y)$ from the continuity equation:

M. I. Kalinin Leningrad Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 43, No. 2, pp. 216-220, August, 1982. Original article submitted June 11, 1981.

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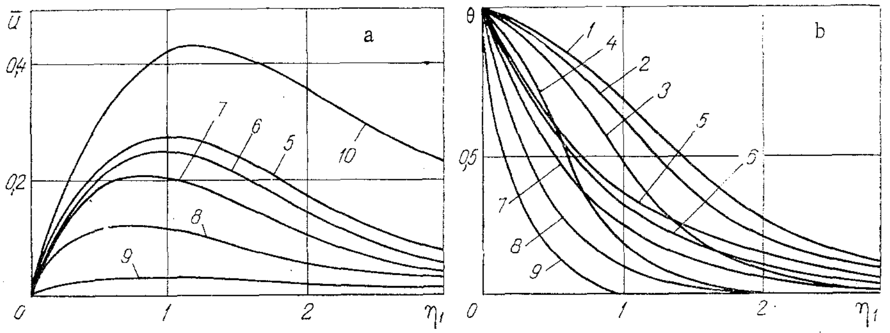

Fig. 1. Distribution of dimensionless velocities (a) and temperatures (b) for $\zeta = 1.1$. $\theta_{w2} = 0.75$; 1) Pr = 0.7; 2) 1; 3) 2; 4) 10; $\theta_{w2} = 1.5$; 5) Pr = 0.7; 6) 1; 7) 2; 8) 10; 9) 100; 10) 0.1.

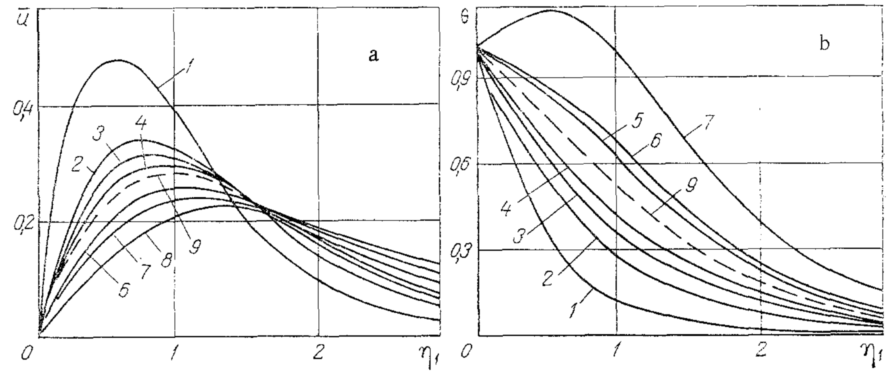

Fig. 2. Profiles of velocities (a) and temperatures (b) in the boundary layer for Pr = 0.7: 1) $\theta_{w2} = 5$; 2) 2; 3) 1.5; 4) 1.25; 5) 0.82; 6) 0.75; 7) 0.5; 8) 0.25; 9) similarity solution; a) $\zeta = 2$; b) $\zeta = 1.1$.

$$\frac{\partial \Psi}{\partial y}\frac{\partial^2 \Psi}{\partial x \partial y} - \frac{\partial \Psi}{\partial x}\frac{\partial^2 \Psi}{\partial y^2} = g\beta\vartheta + \nu\frac{\partial^3 \Psi}{\partial y^3},$$
$$\frac{\partial \Psi}{\partial y}\frac{\partial \vartheta}{\partial x} - \frac{\partial \Psi}{\partial x}\frac{\partial \vartheta}{\partial y} = a\frac{\partial^2 \vartheta}{\partial y^2}. \tag{3}$$

We represent the solution for the stream function and the dimensionless temperature in the form $\Psi(x, y) = 4\nu(\mathrm{Gr}_{x1}/4)^{1/4}F(\eta, \zeta)$, $\theta = \vartheta/\vartheta_{c1}$, $\eta_1 = \frac{y}{x}(\mathrm{Gr}_{x1}/4)^{1/4}$, $\zeta = x/x_0$.

To determine $F(\eta_1, \zeta)$ and $\theta(\eta_1, \zeta)$, we obtain the following differential equations:

$$\frac{\partial^3 F}{\partial \eta_1^3} + 3F\frac{\partial^2 F}{\partial \eta_1^2} - 2\left(\frac{\partial F}{\partial \eta_1}\right)^2 + \theta = 4\zeta\left(\frac{\partial F}{\partial \eta_1}\frac{\partial^2 F}{\partial \zeta \partial \eta_1} - \frac{\partial^2 F}{\partial \eta_1^2}\frac{\partial F}{\partial \zeta}\right), \tag{4}$$
$$\frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial \eta_1^2} + 3F\frac{\partial \theta}{\partial \eta_1} = 4\zeta\left(\frac{\partial F}{\partial \eta_1}\frac{\partial \theta}{\partial \zeta} - \frac{\partial \theta}{\partial \eta_1}\frac{\partial F}{\partial \zeta}\right)$$

with the transformed boundary conditions (2):

$$\zeta \leqslant 1: \frac{\partial F}{\partial \eta_1} = 0,\ F = 0,\ \theta = 1 \text{ for } \eta_1 = 0;\ \frac{\partial F}{\partial \eta_1} = 0,\ \theta = 0 \text{ for } \eta_1 \to \infty;$$
$$\zeta > 1: \frac{\partial F}{\partial \eta_1} = 0,\ F = 0,\ \theta = \theta_{c2} \text{ for } \eta_1 = 0;\ \frac{\partial F}{\partial \eta_1} = 0,\ \theta = 0 \text{ for } \eta_1 \to \infty. \tag{5}$$

Since the temperature variation in the upper part of the plate does not influence the temperature distribution and the distribution of temperatures and velocities in the lower

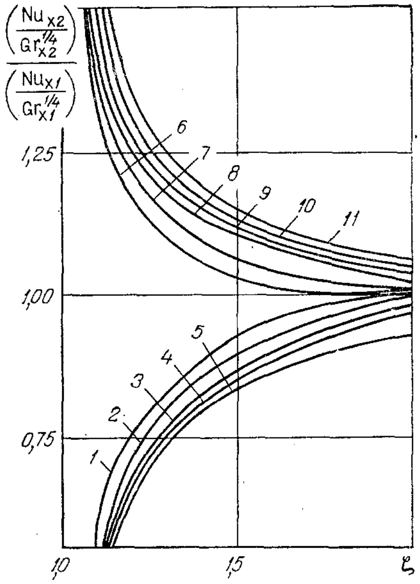

Fig. 3. The dimensionless relative heat transfer coefficient as a function of the parameters θ_{W2} and Pr: θ_{W2} = 0.75; 1) Pr = 100; 2) 10; 3) 2; 4) 0.7; 5) 0.1; θ_{W2} = 1.5; 6) Pr = 100; 7) 10; 8) 2; 9) 1; 10) 0.7; 11) 0.1.

part for $\zeta < 1$, we determine the solution in the lower part of the plate as a similarity problem (the right side of Eq. (4) is zero) [5]. The similarity solution obtained for $x = x_0$ is used as the initial distribution of boundary layer velocities and temperatures. This eliminates the difficulty of assigning the original parameters in the numerical solution of the nonsimilarity problem beyond the discontinuity point. Analytical investigations by perturbation methods show that near the discontinuity point $x = x_0$ the singularity in the distribution of temperatures and heat fluxes has the form $(x-x_0)^{-\frac{1}{3}}$ [6-8].

In the present work numerical solutions of Eqs. (4) and (5) were carried out for Pr = 0.1-100 and θ_{W2} = 0-5. With increase of Pr number in the boundary layer viscous forces predominate and the velocity decreases. The profiles of velocity and temperature become smoother, and the maximum velocity decreases and shifts toward lower values of the similarity variable η (Fig. 1). Here the dynamic boundary layer thickness also decreases.

A decrease of the parameter θ_{W2} leads to a decrease of the maximum dimensionless velocity (Fig. 2a). Because of flow deceleration the velocity profile becomes more smeared. Appreciable peculiarities are observed in the temperature distribution with variation of θ_{W2}. When the temperature ahead of the discontinuity is higher ($\theta_{W2} < 1$), the temperature profile falls off more sharply, the larger is the value of the parameter (Fig. 2b). For $\theta_{W2} = 0$ ($T_{W2} = T_\infty$) the flow near the wall can be regarded as a wall layer with a given integral characteristic for $\zeta = 1$. With increasing distance from the section of the discontinuity the flow becomes homogeneous. No calculations were performed for large values of θ_{W2} because of flow instability, when the upper layers are hotter than the lower layers. In the experiments with $\theta_{W2} > 1.5$ we observed leakage of hot air downward [4]. Evidently, to solve the problem in this case one must use the methods of matched asymptotic expansions [1].

Figure 3 shows a graph of the variation of the heat transfer coefficient with plate height. At the discontinuity section the heat transfer takes infinitely large values ($\alpha \sim (\zeta - 1)^{-1/3}$). As the Pr number increases the heat transfer rapidly becomes equal to its asymptotic value.

Beyond the point of discontinuity in temperatures to calculate the heat transfer one can suggest the approximate relation

$$\left(\frac{\mathrm{Nu}}{\mathrm{Gr}_{x2}^{1/4}}\right)\Big/\left(\frac{\mathrm{Nu}}{\mathrm{Gr}^{1/4}}\right)_{x1}=\theta_{W_2}^{-\frac{5}{4}}\left[\frac{\zeta}{(\zeta-1)\theta_{W_2}+1}\right]^{\frac{1}{4}}\times[1+(\theta_{W_2}^{\frac{3}{2}}-1)\{1-[1+\theta_{W_2}(\zeta-1)]^{-c}\}^{-\frac{1}{3}}], \quad (6)$$

where $c = 0.9016A(\theta_{W_2}^{3/2}-1)^3\theta_{W_2}^{-3/2}(1-\theta_{W2})^{-1}$; $A = [Pr^{1/2}(1 + 3.9932Pr^{1/2} + 2.9584\ Pr)^{1/4}]/[(1 + 1.4903\ Pr^{9/16})^{4/3}]$.

The error in this formula does not exceed 2.5% over the whole range of variation of the parameters.

We obtain the mean value of heat transfer by integrating over the plate height

$$\mathrm{Nu}_l/\mathrm{Nu}_{l1}=1+\frac{3}{4}\theta_{W_2}^{\frac{1}{4}}\int_1^{l/x_0}\zeta^{-\frac{1}{4}}f(\zeta,\ \theta_{W2},\mathrm{Pr})\,d\zeta. \quad (7)$$

In the limiting case $l/x_0 \gg 1$ the heat transfer $\mathrm{Nu}_l/\mathrm{Nu}_{l1}=1+\theta_{W_2}^{\frac{1}{4}}(l/x_0-1)^{3/4}$ is determined from the equation $Nu_{l2} = Nu_{l1}$.

Therefore, the initial section does not affect the heat transfer in the upper part of the plate.

If we use the correlations of [9], to calculate the average heat transfer we can suggest the following formula:

$$\left[\frac{\mathrm{Nu}_l/\mathrm{Nu}_{l1}-1}{\theta_{W_2}^{1/4}(l/x_0-1)^{3/4}}\right]^{\frac{13}{5}}=1+\left[\frac{1.16(1-\theta_{W2})}{\theta_{W_2}(l/x_0-1)^{1/12}A^{1/3}}\right]^{\frac{13}{5}}. \quad (8)$$

For the values of Pr and θ_{W2} investigated the error in Eq. (8) does not exceed 3%.

NOTATION

x, y, longitudinal and transverse coordinates; u, v, components of the velocity vector on the x and y axes; x_0, height of the surface with temperature T_{W1}; l, plate length; T, temperature; $\vartheta = T - T_\infty$, excess temperature; $\theta = (T - T_\infty)/(T_{W1} - T_\infty)$, dimensionless excess temperature; ν, kinematic viscosity; α, diffusivity; g, acceleration due to gravity; β, coefficient of volume expansion; $u_0=\sqrt{g\beta\vartheta_c l}$, velocity scale; $\bar{u} = u/u_0$, dimensionless velocity; $Pr = \nu/\alpha$, Prandtl number; $Gr=g\beta\vartheta_c l^3/\nu^2$, Grashof number; $Nu = \alpha l/\lambda$, Nusselt number. Subscripts: w, values at the wall; ∞, at a large distance from the wall; w1, values at the wall ahead of the discontinuity; w2, the same, after the discontinuity; x, local value.

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